Analysis and Design of Periodic Structures for Microstrip Lines by Using the Coupled Mode Theory

Txema Lopetegi, *Student Member, IEEE*, Miguel A. G. Laso, *Student Member, IEEE*, Maria J. Erro, *Student Member, IEEE*, Mario Sorolla, *Senior Member, IEEE*, and Manfred Thumm, *Fellow, IEEE*

***Abstract*—In this letter, the coupled mode theory is formulated to analyze distributed periodic structures for microstrip lines. After it, several reasonable approximations are introduced giving rise to analytical solutions for the problem. The obtained results, both numerical and analytical, are checked against measurements showing very good agreement. The proposed method is very attractive for the study of these devices since it avoids the time-consuming full-wave electromagnetic simulations customarily employed and provides analytical solutions that are very useful for analysis and synthesis purposes.**

***Index Terms*—Coupled mode theory, microstrip technology, periodic structure (PS).**

I. Introduction

In recent years, there has been an important research effort to develop novel periodic structures (PSs) compatible with planar microwave and millimeter wave circuits. The aim of these PSs is to improve the behavior of circuits and antennas by introducing stopbands to forbid the propagation of electromagnetic waves in the unwanted frequency bands and directions. Due to the similarity between the stopbands in these periodic lattices and the energy bandgaps in crystals (e.g., semiconductors), they are frequently referred to as photonic bandgap (PBG) structures. These novel devices have found very promising applications including the implementation of filters and resonators, the improvement of the efficiency and radiation pattern of antennas, harmonic tuning in power amplifiers, oscillators and mixers, and the suppression of spurious bands in filters [1], [2]. However, the analysis and design of these structures has been laborious because it has required full-wave electromagnetic simulations, and a trial and error design method. To surpass these difficulties, the use of the coupled mode theory is proposed in this letter.

II. Application of the Coupled Mode Theory to Periodic Structures for Microstrip Lines

In order to formulate an accurate coupled mode theory suitable for microwave devices, the cross section method will be employed. The basic idea of this method is that the electromagnetic fields at any cross section of a nonuniform waveguide can be represented as a superposition of the different modes (including their forward and backward traveling waves), corresponding to a uniform auxiliary waveguide that has the same cross section with identical distribution of ε and μ. The coefficients of this superposition satisfy a system of first-order ordinary differential equations, that turn into integro-differential equations for open waveguides, and are known as the coupled mode equations [3]–[5].

Focusing our study on the novel PSs for microstrip lines implemented in the ground plane, and also known in the literature as PBGs, the system of coupled mode equations can be notably simplified by realizing several very reasonable approximations. In the first place, we are going to neglect the coupling of energy to the modes of the continuous spectrum. This can be done because the energy of these modes is primarily radiated, and since the devices that we are going to study have very little radiation losses in the frequency band of interest, the error involved in the approximation will be small. Regarding the modes of the discrete part of the spectrum, for the frequency band of interest, we will only have the quasi-TEM mode for the sections of pure microstrip line, and a quasi-microstrip mode together with a quasi-slot mode for the sections of microstrip line with slot in the ground plane that appear due to the introduction of the periodic structure. Since the excitation of the devices under study will be done through a pure microstrip line, the excitation mode will be always the quasi-TEM mode of the microstrip line. This mode has very high correlation with the quasi-microstrip mode (actually it can be seen as the same mode), and very low correlation with the quasi-slot mode, so we can approximate neglecting the excitation of the quasi-slot mode and assuming that only the quasi-microstrip mode is excited. These approximations lead us to assume single mode operation in our PS for microstrip lines, and simplify the coupled mode equations obtaining

$$\frac{da^+}{dz} = -j \cdot \beta \cdot a^+ + K \cdot a^- \tag{1}$$

$$\frac{da^-}{dz} = K \cdot a^+ + j \cdot \beta \cdot a^- \tag{2}$$

with:

$$\hat{\vec{E}} = a^+ \cdot \vec{E}^+ + a^- \cdot \vec{E}^-; \quad \hat{\vec{H}} = a^+ \cdot \vec{H}^+ + a^- \cdot \vec{H}^- \tag{3}$$

$$N^+ = \iint_S \vec{E}^+ \times \vec{H}^+ \cdot d\vec{S} = -N^- = -\iint_S \vec{E}^- \times \vec{H}^- \cdot d\vec{S} \tag{4}$$

$$\beta = \frac{2 \cdot \pi \cdot f}{c} \cdot \sqrt{\varepsilon_{eff}}; \qquad c = 3 \cdot 10^8 \text{ m/s} \tag{5}$$

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T. Lopetegi, M. A. G. Laso, M. J. Erro, and M. Sorolla are with the Electrical Engineering Department, Public University of Navarre, Pamplona, Spain.

M. Thumm is with the IHE, University of Karlsruhe, and the IHM, Research Center Karlsruhe, Eggenstein-Leopoldshafen, Germany.

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$$K = \frac{1}{2N^+} \cdot \iint_S \left(\vec{E}^+ \times \frac{d\vec{H}^-}{dz} + \frac{d\vec{H}^+}{dz} \times \vec{E}^- \right) \cdot d\vec{S} + \frac{1}{2N^+} \cdot \frac{dN^-}{dz} \tag{6}$$

being $\hat{\vec{E}}$, $\hat{\vec{H}}$ the total electric and magnetic field present in the structure; z the direction of propagation; $\vec{E}^+, \vec{H}^+, \vec{E}^-, \vec{H}^-$, the (x, y) dependent part of the electric and magnetic fields of the forward $(+)$ and backward $(-)$ traveling waves corresponding to the microstrip (or quasi-microstrip) mode of the auxiliary uniform waveguide associated to the cross section of interest (it varies with z). It is important to note that these last fields depend, finally, also on z, due to the variation that the cross section (and hence the auxiliary uniform waveguide) suffers with z. The factors N^+, N^- are the normalizations taken for the fields of the mode, and S the surface of the cross section. To simplify the expressions, we are going to assume for the rest of this letter that the normalizations do not vary with the cross section (constant with z). β and ε_{eff} are the phase constant and effective dielectric constant of the mode in the auxiliary uniform waveguide associated with the cross section of interest (it varies with z). f is the frequency of operation. K is the coupling coefficient of the structure. a^+, a^- are only function of z and f, and can be seen as the complex amplitude of the forward $(+)$ and backward $(-)$ traveling waves in the nonuniform waveguide.

The expression for the coupling coefficient can be reformulated with the introduction of the characteristic impedance of the mode, Z_0, obtaining

$$K = -\frac{1}{2} \cdot \frac{1}{Z_0} \cdot \frac{dZ_0}{dz} + \frac{1}{N^+} \cdot \iint_S \left(\vec{e}^+ \times \frac{d\vec{h}^+}{dz} \right) \cdot d\vec{S} \tag{7}$$

with

$$\vec{e}^+ = \frac{1}{\sqrt{Z_0}} \cdot \vec{E}^+; \qquad \vec{h}^+ = \sqrt{Z_0} \cdot \vec{H}^+. \tag{8}$$

The part corresponding to the fields of (7) can be neglected if the Z_0 of the mode characterizes adequately the propagation of the mode along the waveguide in reflection terms. This can be achieved for the microstrip mode calculating Z_0 by using the power and the current.

To finish this section, it is important to highlight that although the periodic structure employed can be two-dimensional, due to the high confinement of the fields around the conductor strip of the microstrip line, we only take advantage of the periodicity along the conductor strip direction (z), and, as a result, the operation and modeling of the device is one-dimensional (1-D) effective.

III. Approximate Analytical Solution

Due to the fact that the PS for microstrip lines will be periodic along the z direction with period Λ, the coupling coefficient $K(z)$ will be periodic with the same period, and, hence, it will be amenable to be expanded in a Fourier series

$$K(z) = \sum_{n=-\infty}^{n=\infty} K_n \cdot e^{j\cdot(2\pi/\Lambda)\cdot n \cdot z}; \qquad K_n = \frac{1}{\Lambda} \cdot \int_\Lambda K(z) \cdot e^{-j\cdot(2\pi/\Lambda)\cdot n \cdot z} \cdot dz. \tag{9}$$

To obtain the analytical solutions, we are going to take the following approximations:

- We approximate β (and hence ε_{eff}) as a variable constant with z. This approximation involves the election of an "averaged" ε_{eff}. The most adequate value for it is

$$\varepsilon_{eff} = \left(\frac{1}{\Lambda} \cdot \int_0^\Lambda \sqrt{\varepsilon_{eff}(z)} \cdot dz \right)^2. \tag{10}$$

- We neglect the terms that include complex exponentials that vary very quickly with z, keeping the terms with complex exponentials that vary slowly with z. This can be done for the range of frequencies placed around the maximum reflection frequency of the rejected band (band gap) of interest. As it can be seen, this approximation involves the election of a rejected band of interest around which the analytical solution will be valid. Each K_n of the Fourier series will correspond to the rejected band of index n, being the index of the first rejected band $n = 1$. When the rejected band of interest is selected (by fixing n), the corresponding K_n must be taken and its value is given to the κ parameter

$$\kappa = -j \cdot K_n. \tag{11}$$

With these approximations, the coupled mode equations can be reformulated in a simplified form that has an analytical solution. We are going to continue centering our study in a PS for microstrip lines of finite length, L. From the analytical solutions for a^+, a^- (obtained assuming that the output port is matched, $a^-_{out} = 0$) of its simplified coupled mode equations, the following expressions for the S_{11} and S_{21} of the device can be obtained [6]:

$$S_{11} = \left.\frac{a^-_{in}}{a^+_{in}}\right|_{a^-_{out}=0} = \frac{-j \cdot \kappa \cdot \sinh(\gamma \cdot L)}{j \cdot \Delta\beta \cdot \sinh(\gamma \cdot L) + \gamma \cdot \cosh(\gamma \cdot L)} \tag{12}$$

$$S_{21} = \left.\frac{a^+_{out}}{a^+_{in}}\right|_{a^-_{out}=0} = \frac{j \cdot \gamma \cdot e^{-j\cdot(\pi/\Lambda)\cdot n \cdot L}}{-\Delta\beta \cdot \sinh(\gamma \cdot L) + j \cdot \gamma \cdot \cosh(\gamma \cdot L)} \tag{13}$$

with

$$\gamma = +\sqrt{|\kappa|^2 - (\Delta\beta)^2}; \qquad \Delta\beta = \beta - \frac{\pi}{\Lambda} \cdot n. \tag{14}$$

Using these expressions, it can be seen that the bandwidth (between zeros) of the obtained rejected band, the frequency of maximum attenuation and the value of this maximum attenuation (minimum S_{21}), are given by [6]

$$BW = \frac{c \cdot |\kappa|}{\pi \cdot \sqrt{\varepsilon_{eff}}} \cdot \sqrt{1 + \left(\frac{\pi}{|\kappa| \cdot L} \right)^2} \tag{15}$$

$$f_{\max} = \frac{c \cdot n}{\sqrt{\varepsilon_{eff}} \cdot \Lambda \cdot 2}; \quad |S_{21}|_{\min} = \operatorname{sech}(|k| \cdot L). \tag{16}$$

From the analytical solutions for a^+, a^-, the transmission matrix of the device with length L can be also obtained [7].

$$A_{11} = A_{22}^* = \left(\cosh(\gamma \cdot L) + \frac{j \cdot \Delta\beta \cdot L \cdot \sinh(\gamma \cdot L)}{\gamma \cdot L}\right) \cdot e^{j\cdot(\pi/\Lambda)\cdot n\cdot L} \tag{17}$$

$$A_{21} = A_{12}^* = \frac{-j \cdot \kappa \cdot L \cdot \sinh(\gamma \cdot L)}{\gamma \cdot L} \cdot e^{j\cdot(\pi/\Lambda)\cdot n\cdot L}. \tag{18}$$

The transmission matrices allow the analytical study of nonuniform PSs obtained by cascading uniform ones. In particular, the presence of defects (e.g., missing periods) can be modeled.

IV. Numerical Results and Measurements

Using the theory and expressions developed in the previous sections, we are going to analyze the distributed PS for microstrip lines proposed in [8]. It is obtained by drilling a periodic pattern of circles, arranged in a square lattice, in the ground plane of a microstrip line. The parameters that characterize the PS are the period (distance between the center of adjacent circles, Λ), and the radius of the circles (given through the radius to period ratio, r/Λ). The prototype will be implemented in a Rogers Duroid 6010 substrate, with dielectric constant 10.2, and thickness 50 mils. The microstrip line will have a strip conductor width $w = 1.2$ mm, corresponding to a characteristic impedance of 50 Ω. The number of periods of the PSs will be nine, with period $\Lambda = 14.1$ mm and $r/\Lambda = 0.25$. This prototype has been designed by scaling the one presented in [8] in order to be measured with our HP 8753D vector network analyzer. The device has been simulated using the coupled mode equations proposed in Section II. The phase constant β (and hence ε_{eff}), and the characteristic impedance Z_0 of the auxiliary uniform waveguides have been previously calculated for a set of them (microstrip lines with slots of different widths in the ground plane) and the exact values needed to solve the coupled mode equations are obtained by interpolation. The coupling coefficient has been calculated by (7), and neglecting the part of the fields. The approximate analytical solutions given in Section III have also been calculated for the first rejected band ($n = 1$). In Fig. 1, the S_{21} parameter for the prototype is presented including the value obtained from coupled mode theory simulation, the approximate analytical solution (13) and the measurement results. An excellent agreement is obtained between all of them. The frequency of maximum attenuation, value of this maximum attenuation (minimum S_{21}), and the bandwidth (between zeros) of the rejected band in the measurement results are $f_{\max} = 4.45$ GHz, $|S_{21}|_{\min} = -41.20$ dB, $BW = 1.91$ GHz. On the other hand, the values predicted for these parameters by the (15), (16) are $f_{\max} = 4.51$ GHz, $|S_{21}|_{\min} = -41.25$ dB and $BW = 2.0$ GHz. The agreement between the analytical and the measured values is very good. The prototype of defected PS resonator designed in [2] has been analogously studied using time transmission matrices for the analytical calculations. A very good agreement is again obtained between the numerical, analytical, and measured results.

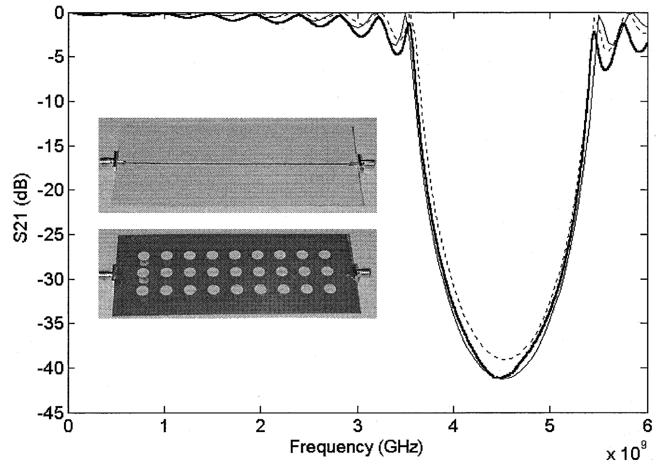

Fig. 1. S_{21} parameter for the microstrip line with PS prototype obtained by coupled mode theory simulation (dashed line), analytical expressions (thin solid line), and measurement (thick solid line).

V. Conclusion

In this paper, the use of the coupled mode theory to analyze distributed PSs for microstrip lines has been proposed and successfully tested. The method has been applied to a particular PS for microstrip lines, but it can be easily extended to other PSs and to other planar waveguides. Even the extension to more complicated multimode waveguides is feasible since, for the frequency range around the frequency of maximum attenuation (where the so called resonant coupling between the forward and backward traveling waves of interest takes place), the coupling to other modes can be customarily neglected [3]. The coupled mode theory provides very useful insight into the operation of the structure, and the analytical solutions obtained constitute an excellent set of tools for the analysis and design of these devices.

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